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TRIGONOMETRÍA

DIRIGIDA

SEMANA 16

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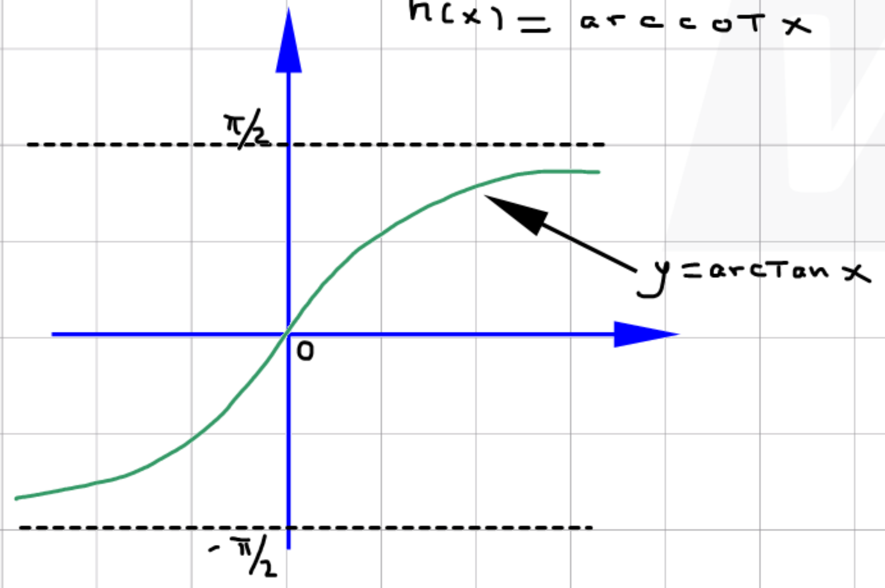
PROBLEMA 1

El dominio de la función f definida por:

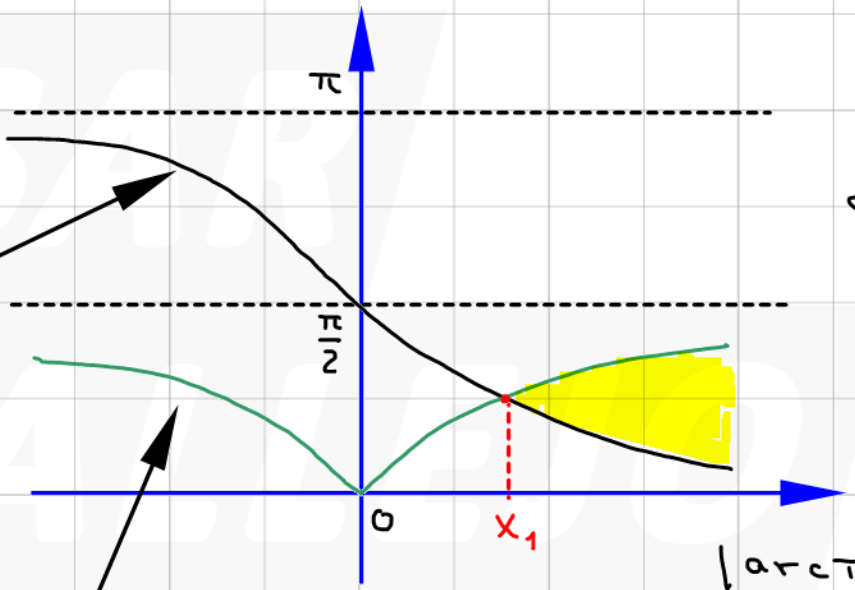
$f(x)=\sqrt{|\arctan(x)|-|\operatorname{arccot}(x)|}$, es:

A) $[-1;+\infty\rangle$ B) $\langle-\infty;-1]$ C) $[1;+\infty\rangle$
D) $\langle-\infty;1]$ E) $[-1;1]$



Resolución

- $\arctan x \longrightarrow x\in\mathbb{R}$
- $\operatorname{arccot} x \longrightarrow x\in\mathbb{R}$
- $\underbrace{|\arctan x|}_{g(x)}-\underbrace{|\operatorname{arccot} x|}_{+}\geq 0 \longrightarrow |\arctan x| \geq \underbrace{\operatorname{arccot} x}_{h(x)}$



$g(x)\geq h(x)$

$\longrightarrow x\in[x_1;+\infty\rangle$

$\longrightarrow \operatorname{Dom} f=[x_1;+\infty\rangle$

$g(x_1)=h(x_1)$

$\underbrace{|\arctan x_1|}_{+}=\operatorname{arccot} x_1$

$\arctan x_1=\operatorname{arccot} x_1$

$\longrightarrow x_1=1$

$\therefore \operatorname{Dom} f=[1;+\infty\rangle$

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PROBLEMA 2

Calcule el valor de

$$\cos\left(\operatorname{arccot}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)-\operatorname{arccot}(\sqrt{2})\right)$$

A) $\frac{\sqrt{2}}{4}$

B) $\frac{\sqrt{3}}{2}$

C) $\frac{\sqrt{2}}{2}$

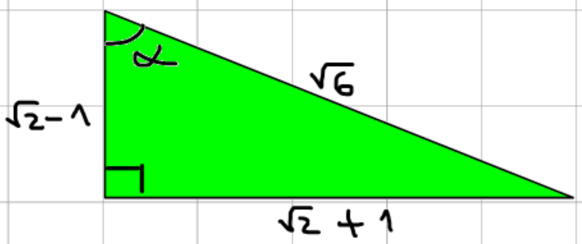
D) $\frac{1}{2}$

E) 1

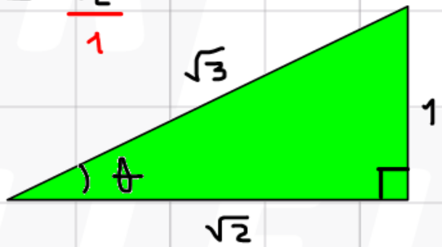
Resolución

$$K=\cos\Big(\underbrace{\operatorname{arccot}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}_{\alpha}-\underbrace{\operatorname{arccot}\sqrt{2}}_{\theta}\Big)$$

$$\cot\alpha=\frac{\sqrt{2}-1}{\sqrt{2}+1}$$



$$\cot\theta=\frac{\sqrt{2}}{1}$$



Luego

$$K=\cos(\alpha-\theta)$$

$$K=\cos\alpha\cos\theta+\operatorname{sen}\alpha\operatorname{sen}\theta$$

$$K=\left(\frac{\sqrt{2}-1}{\sqrt{6}}\right)\cdot\frac{\sqrt{2}}{\sqrt{3}}+\left(\frac{\sqrt{2}+1}{\sqrt{6}}\right)\cdot\frac{1}{\sqrt{3}}=\frac{2-\sqrt{2}}{3\sqrt{2}}+\frac{\sqrt{2}+1}{3\sqrt{2}}$$

$$\therefore K=\frac{\sqrt{2}}{2}$$

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PROBLEMA 3

Determine el rango de la función f definida por:

$$f(x)=6\operatorname{arc\,cot}\left(\frac{2\sqrt{3}x^4}{1+x^8}\right)+\pi$$

A) $\langle 2\pi;\, 4\pi\rangle$ B) $[3\pi;\, 4\pi]$ C) $\langle 3\pi;\, 4\pi\rangle$

D) $[2\pi;\, 4\pi]$ E) $\langle 2\pi;\, 5\pi\rangle$

Resolución

- si $x=0$, entonces

$$f(0)=6\underbrace{\operatorname{arccot}(0)}_{\frac{\pi}{2}}+\pi$$

$$\longrightarrow f(0)=4\pi \text{ --------- (I)}$$

- si $x\neq 0$, entonces

$$f(x)=6\operatorname{arccot}\left(\frac{2\sqrt{3}}{x^4+\frac{1}{x^4}}\right)+\pi \quad \frac{2\sqrt{3}}{x^4+\frac{1}{x^4}} \dashrightarrow E$$

$$\longrightarrow y=6\operatorname{arccot}(E)+\pi$$

$$x^4+\frac{1}{x^4}\geq 2 \longrightarrow 0<\frac{1}{x^4+\frac{1}{x^4}}\leq\frac{1}{2}$$

$$0<\frac{2\sqrt{3}}{x^4+\frac{1}{x^4}}\leq\sqrt{3} \longrightarrow 0<E\leq\sqrt{3}$$

La función arco cotangente es decreciente, entonces

$$\operatorname{arccot}(0)>\operatorname{arccot}E\geq\operatorname{arccot}(\sqrt{3}) \longrightarrow \frac{\pi}{2}>\operatorname{arccot}E\geq\frac{\pi}{6}$$

$$3\pi>6\operatorname{arccot}E\geq\pi \longrightarrow 4\pi>f(x)\geq 2\pi \text{ ----- (II)}$$

De (I) $\cup$ (II):

$$\operatorname{Ran}f=[2\pi;\,4\pi]$$

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PROBLEMA 4

Calcule el rango de la función *f* definida por

$$f(x)=\sqrt{\text{arcsen}(x)-\text{arctan}(x)}+\sqrt{x^2-1}$$

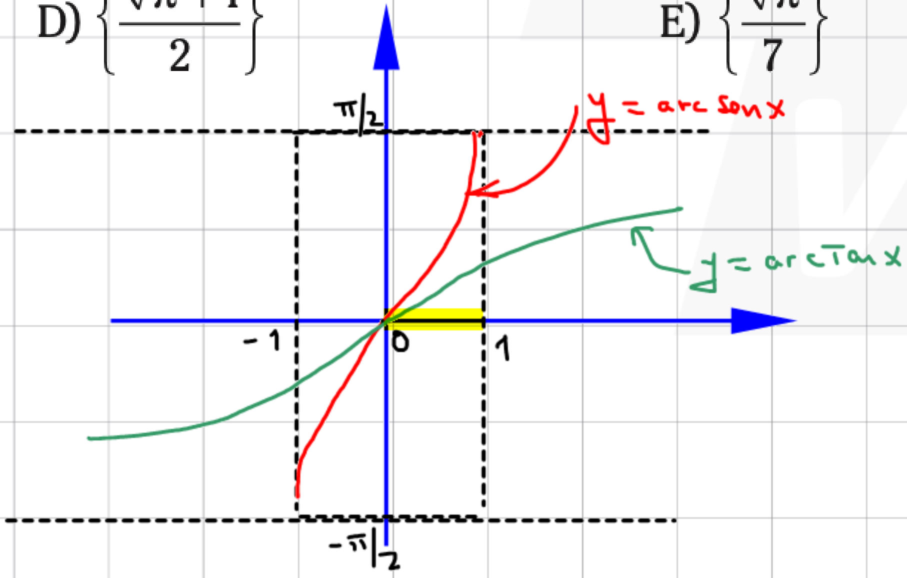
A) $\{\pi\}$

B) $\left\{\frac{\sqrt{\pi}}{2}\right\}$

C) $\left\{\frac{\sqrt{\pi}}{3}\right\}$

D) $\left\{\frac{\sqrt{\pi}+1}{2}\right\}$

E) $\left\{\frac{\sqrt{\pi}}{7}\right\}$



Resolución

- $\text{arc sen}\,x \longrightarrow -1\leq x\leq 1$ ---------- (I)
- $\text{arcTan}\,x \longrightarrow x\in\mathbb{R}$ ---------- (II)
- $x^2-1\geq 0 \longrightarrow x\leq -1 \vee x\geq 1$ ---------- (III)
- $\text{arcsen}\,x-\text{arcTan}\,x\geq 0$

  $\text{arcsen}\,x\geq\text{arcTan}\,x \longrightarrow 0\leq x\leq 1$ ---------- (IV)

Intersectamos (I), (II), (III) y (IV)

$$x=1$$

$$\longrightarrow \text{Dom}f=\{1\}$$

Luego

$$f(1)=\sqrt{\underbrace{\text{arcsen}(1)}_{\frac{\pi}{2}}-\underbrace{\text{arcTan}(1)}_{\frac{\pi}{4}}}+\sqrt{1^2-1}=\frac{\sqrt{\pi}}{2}$$

$$\therefore\ \text{Ran}f=\{f(1)\}=\left\{\frac{\sqrt{\pi}}{2}\right\}$$

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PROBLEMA 5

Determine el máximo valor de la siguiente función:

$f(x)=2\arcsen x+4\arctan x+\pi$

A) 3π B) 2π C) π
D) 4π E) 6π

Resolución

$f(x)=2\underbrace{\arcsen x}_{-1\leq x\leq 1}+4\underbrace{\arctan x}_{x\in\mathbb{R}}+\pi$

$\rightarrow \text{Dom} f=[-1;1]$

f es creciente:

$-1\leq x\leq 1 \longrightarrow f(-1)\leq f(x)\leq f(1)$

$2\underbrace{\arcsen(-1)}_{-\frac{\pi}{2}}+4\underbrace{\arctan(-1)}_{-\frac{\pi}{4}}+\pi\leq f(x)\leq 2\underbrace{\arcsen(1)}_{\frac{\pi}{2}}+4\underbrace{\arctan(1)}_{\frac{\pi}{4}}+\pi$

$\longrightarrow -\pi\leq f(x)\leq 3\pi$

$\therefore f_{máx}=3\pi$

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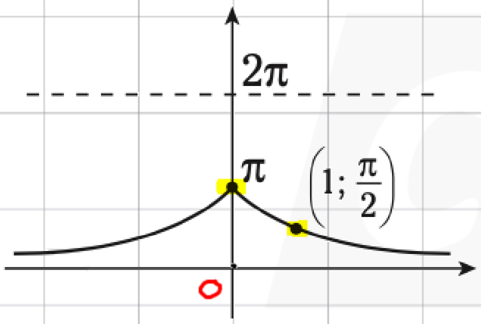
PROBLEMA 6

En el gráfico se tiene la función definida mediante $f(x)=A\operatorname{arccot}(B|x|)$, siendo A y B cantidades positivas entonces el valor de $A+B$ es



A) 2 B) 3 C) 4
D) 5 E) 6

Resolución

$f(x)=A\operatorname{arccot}(B|x|)\ ;\ A>0,\ B>0$

- $f(0)=\pi \longrightarrow A\underbrace{\operatorname{arccot}(0)}_{\frac{\pi}{2}}=\pi$

$\longrightarrow A=2$

- $f(1)=\frac{\pi}{2} \longrightarrow 2\operatorname{arccot}(B)=\frac{\pi}{2}$

$\operatorname{arccot}(B)=\frac{\pi}{4}$

$B=\cot\frac{\pi}{4} \longrightarrow B=1$

$\therefore A+B=3$

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PROBLEMA 7

Determine el rango de la función

$f(x)=\arctan(|\operatorname{sen}x|+|\cos x|)$

A) $\left[1,\ \arctan\left(\frac{\sqrt{2}}{4}\right)\right]$

B) $\left[0,\ \arctan\left(\frac{\pi\sqrt{2}}{4}\right)\right]$

C) $\left[\frac{\pi}{4},\ \arctan(\sqrt{2})\right]$

D) $\left[0,\ \arctan\left(\frac{\pi\sqrt{2}}{2}\right)\right]$

E) $\left[1,\ \frac{\pi\sqrt{2}}{4}\right]$

Resolución

$f(x) = \arctan(\underbrace{|\operatorname{sen}x|+|\cos x|}_{K}) \longrightarrow y = \arctan(K)$

- Por existencia del arcoTangente: $K \in \mathbb{R}$ --------- (I)
- $K = |\operatorname{sen}x| + |\cos x|$; $K > 0$

$K^2 = 1 + \underbrace{|\operatorname{sen}2x|}_{0\le\ \ \le 1} \longrightarrow 1 \le K^2 \le 2$

$\longrightarrow 1 \le K \le \sqrt{2}$ -------- (II)

De (I) y (II): $1 \le K \le \sqrt{2}$

El arco Tangente es creciente, entonces

$\arctan(1) \le \arctan(K) \le \arctan(\sqrt{2})$

$\frac{\pi}{4} \le f(x) \le \arctan\sqrt{2}$

$\therefore \operatorname{Ran} f = \left[\frac{\pi}{4};\ \arctan\sqrt{2}\right]$

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GRACIAS

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